

On the force between two metallic plates of a gripper immersed in a nonpolar fluid

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Abstract

We analyse, as a function on the temperature T and the chemical potential μ , the total force $F_{tot}(T, \mu, L)$ between two metallic plates of a gripper separated at a distance L from each other and immersed in a nonpolar fluid which can be liquid, or gas. In our approach we take into account the direct substrate-substrate van der Waals interaction, the van der Waals interactions between the molecules of the fluid with the other molecules of the fluid as well as with the constituent elements of the substrate, and the interaction between the plates generated by the fluctuations of the density of the fluid (i.e., the Casimir force). We suppose that both plates are equal and strongly prefer the liquid phase of the fluid. Under such boundary conditions both the direct plate-plate van der Waals interaction, as well as the Casimir force, are forces of attraction of the plates toward each other. In the phase space (temperature, chemical potential), we identify the regions where the net interaction force is the strongest. It turns out that these regions are close to the bulk critical point of the fluid ($T = T_c, \mu = \mu_c$), and near the so-called capillary condensation regime $T < T_c, (L/a)(\Delta\mu/k_B T) = O(1)$, with $\Delta\mu = \mu - \mu_c < 0$ and a the characteristic distance between the molecules of the fluid. These regions shall be avoided in order to prevent sticking of the plates of the gripper on each other.

Keywords: grippers, van der Waals forces, Casimir effect, thin films

1. Introduction

Handling and fixing of micro parts reliably and precisely is the main bottleneck in micro assembly and is far from being solved today. Further handling and fixing strategies must be developed taking into account the forces appearing in small distances between the gripper's plates and the micro object or between the object and a surface to taken off or to be placed on. Also the environmental influences are usually not mastered yet and they have to be studied.

Tools for assisting the robot in micro assembly tasks are not available today. Components such as micro part feeders and miniature grippers for micro objects also put some problems of interaction in small distances with micro objects that should be studied in order to find ways to control these operations.

In this article we study the force between two metallic plates of a gripper immersed in nonpolar fluid.

If a fluid is confined by parallel plates at a distance L (see Fig. 1) and is in contact with a particle reservoir with a chemical potential μ and temperature T , the grand canonical potential $\Omega_{ex}(T, \mu, L)$ of the fluid in excess to its bulk value $AL\omega_{bulk}(T, \mu)$ depends on L . Then the effective force F_{tot} between the plates normalized per cross sectional area A is

$$F_{tot}(T, \mu, L) = -k_B T \frac{\partial \Omega_{ex}(T, \mu, L)}{\partial L}, \quad (1)$$

where k_B is the Boltzmann's constant,

$$\begin{aligned} \omega_{ex}(T, \mu, L) &= \omega(T, \mu, L) - L\omega_{bulk}(T, \mu) \\ &= \Omega_{ex}(T, \mu, L) / A \end{aligned} \quad (2)$$

is the excess grand canonical potential per cross sectional area A , $\Omega(T, \mu, L) = A\omega(T, \mu, L)$ is the total grand canonical potential, and $\omega_{bulk}(T, \mu)$ is the density of the bulk grand canonical potential. Besides

temperature T , chemical potential μ , and film thickness L , the force also depends on which boundary conditions the surfaces impose on the system. The order near the surfaces can be either reduced or -- which is the generic case for liquids confined by solid substrates -- increased due to effective surface fields generated by the confinement. The latter case is known as (+,+) boundary conditions (for a more precise definition see below). For this case, the schematic phase diagram of a fluid film with thickness L is shown in Fig. 2.

In order to calculate the effective force between the

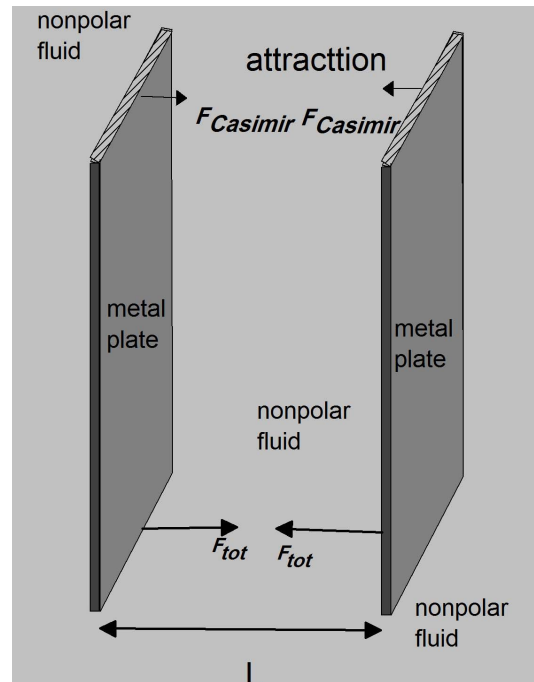


Fig. 1. The total and the Casimir force acting on the plates of a gripper. Note that they both are forces of attraction, i.e. the Casimir force causes stronger attraction between the plates.

plates we consider long-ranged pair interactions between the fluid particles decaying with distances r between each other asymptotically $\sim J^l r^{-6}$ and long-ranged substrate potentials $\sim J^{l,s} z^{-3}$ acting on fluid particles at a distance z from the flat surface of a semi-infinite substrate. As it has been shown in [1] in the case of d -dimensional systems, in which the interaction decays asymptotically with the distance $\sim r^{-d-\sigma}$ (with $\sigma > 2$), the effective total force $F_{tot}(T, \mu, L)$ between the plates can be written in the form:

$$F_{tot}(T, \mu, L) \simeq (\sigma - 1) H_A(T, \mu) L^{-\sigma} \xi_0^{\sigma-d} + k_B T L^{-d} X \left[\frac{L}{\xi_t}, \frac{L}{\xi_\mu}, l \left(\frac{L}{\xi_0} \right)^{-\omega_l}, s \left(\frac{L}{\xi_0} \right)^{-\omega_s}, g_\omega \left(\frac{L}{\xi_0} \right)^{-\omega} \right]. \quad (3)$$

In the first term of the above equation X is dimensionless, universal scaling function, which takes into account the part of the force that is due to the fluctuations of the density of the fluid between the plates, i.e. the Casimir force [2-7], ξ_t is the bulk correlation length, i.e. $\xi_t = \xi(t \rightarrow \pm 0, \Delta\mu = 0) \simeq \xi_0^\pm |t|^{-\nu}$ at bulk coexistence $\mu = \mu_c$ and for $t = (T - T_c)/T_c \rightarrow \pm 0$, while $\xi_\mu = \xi(t = 0, \Delta\mu \rightarrow 0) = \xi_{0,\mu} |\beta_c \Delta\mu|^{-\nu/\Delta}$ is the bulk correlation length at the critical temperature $T = T_c$ with $\beta_c = (k_B T_c)^{-1}$. For $T > T_c$ and $\Delta\mu = 0$ one has $\xi_0 = \xi_0^+$, while for $T < T_c$ and $\Delta\mu = 0$ one has $\xi_0 = \xi_0^-$ with the ratio ξ_0^+ / ξ_0^- being universal; $\xi_{0,\mu}$ is the same for $\Delta\mu \rightarrow +0$ and $\Delta\mu \rightarrow -0$.

The second term in Eq. (3) stems from the free energy contribution $H_A L^{-(\sigma-1)}$, where

$$H_A(T, \mu) = A_{l,s}(T, \mu) + A_l(T, \mu) + A_s(T) \quad (4)$$

with the so-called Hamaker constants $A_{l,s}$, A_l and A_s . Here A_l represents that part of the Hamaker constant which is generated by the long-ranged part of the fluid-fluid interactions, i.e., the dispersion interaction, A_s is the part due to the direct long-ranged interactions between the two substrates, and $A_{l,s}$ is the corresponding term generated by the long-ranged tails of the substrate potentials acting on the fluid particles (the explicit dependence of $A_{l,s}$, A_l and A_s on the parameters of the model will be presented below). In Eq. (3) $\omega \simeq 0.81$ (with $d=\sigma=3$) is the standard Wegner's correction-to-scaling exponent for short-ranged systems, while $\omega_l = \sigma - (2 - \eta)$ and $\omega_s = \sigma - (d + 2 - \eta)/2$ are the correction to scaling exponents due to the long-ranged tails of the fluid-fluid and substrate-fluid interactions, respectively. For realistic systems, i.e., for the "genuine" non-retarded van der Waals interaction, which governs the nonpolar fluids, one has $d=\sigma=3$ and, then,

$$\eta = 0.034, \beta = 0.329, \nu = 0.631, \Delta = 1.567, \omega = 0.81, \omega_l = 1.03, \omega_s = 0.52. \quad (5)$$

In Eq. (3) the factor g_ω is the dimensionless scaling field associated with the Wegner-type corrections, while

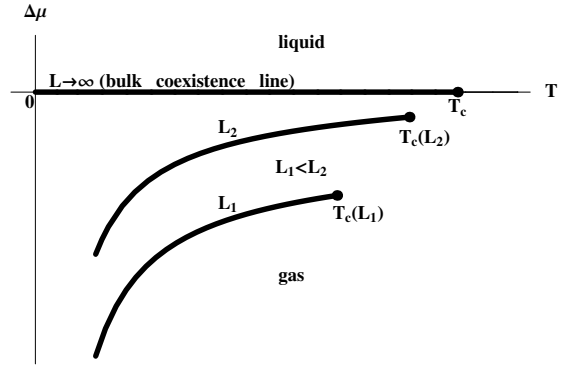


Fig. 2. The phase diagram of a thin fluid film bounded by metal plates, which strongly prefer the liquid phase of the fluid. The bent gas-liquid coexistence curves correspond to capillary condensation transitions for $L=L_1$ and $L=L_2$ with $L_1 < L_2$.

l and s are dimensionless nonuniversal coupling constants with l being proportional to the strength J^l of the long-ranged tail in the fluid-fluid interaction and s being proportional to the contrast between the substrate potential and the fluid-fluid interaction integrated over a half-space (see below).

As it has been demonstrated in [1], when $L \gg L_{crit}$, with

$$L_{crit} = \xi_0 (2^{\sigma+1} |s|)^{\nu/\beta}, \quad (6)$$

one can expand the scaling function X in Eq. (3) which leads to:

$$X \simeq X^{sr} \left[\frac{L}{\xi_t}, \frac{L}{\xi_\mu} \right] + s \left(\frac{L}{\xi_0} \right)^{-\omega_s} X_s^{lr} \left[\frac{L}{\xi_t}, \frac{L}{\xi_\mu} \right] + l \left(\frac{L}{\xi_0} \right)^{-\omega_l} X_l^{lr} \left[\frac{L}{\xi_t}, \frac{L}{\xi_\mu} \right] + g_\omega \left(\frac{L}{\xi_0} \right)^{-\omega} X_\omega^{sr} \left[\frac{L}{\xi_t}, \frac{L}{\xi_\mu} \right]. \quad (7)$$

One concludes that the leading behaviour of the scaling function X very near the bulk critical point $L/\xi_t = 0$, $L/\xi_\mu = 0$, i.e. when $L/\xi_t \ll 1$ and $L/\xi_\mu \ll 1$, can then be determined by only taking into account the short ranged part of the fluid-fluid and the substrate-fluid interactions, i.e. will be given by the function $X^{sr} [L/\xi_t, L/\xi_\mu]$. This normally greatly simplifies the analytical and the numerical difficulties of the calculation. If $L \leq L_{crit}$, however, one has to take the long-ranged tails of the interactions into account in the whole "critical region of the finite system", i.e. when $L/\xi_t = O(1)$ and $L/\xi_\mu = O(1)$. Having in mind that for the most fluids ξ_0 typically is of the order of 3 Å one has

$$L_{crit} \simeq 60 |s|^{1.918} \text{ nm}. \quad (8)$$

As discussed in [1] for a variety of systems $s \in [1, 2]$. However, for some systems such as ^3He or ^4He films near their bulk liquid-gas critical point and confined by Au substrate [8] s can be as large as 4. Thus, one has that L_{crit} , depending on the system, is between 60 nm and 1 μm . In the current article, on the example of a gripper immersed in a typical nonpolar fluid system with $L=100-150$ Å, $s=1$, and $l=0.3$, we will

calculate the total force of attraction between the gripper plates, taking into account the long-ranged tails of the interactions involved and the fluctuations of the density of the fluid.

The structure of the article is as follows. In the next section, we define the model in which the concrete analytical and numerical calculations are performed. Section 3 will present the obtained results for the T and μ dependence of the force. The article closes with a discussion presented in Section 4.

2. The model

Within the density functional approach for inhomogeneous fluids confined between two parallel flat plates at a distance L the grand canonical functional $\Omega[\rho(\mathbf{r})]$ of the fluid:

$$\Omega[\rho(\mathbf{r})] = \int f_{HS}[\rho(\mathbf{r})] d^3\mathbf{r} + \int V(z)\rho(\mathbf{r}) d^3\mathbf{r} + \frac{1}{2} \iint \rho(\mathbf{r})w(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') d^3\mathbf{r} d^3\mathbf{r}' - \mu \int \rho(\mathbf{r}) d^3\mathbf{r} \quad (9)$$

has to be minimized with respect to the local number density $\rho(\mathbf{r})$ [9]. The plates exert a substrate potential $V(z)$ with z as the normal distance from one wall. For an individual wall $V(z \rightarrow \infty) \sim z^{-\sigma}$ where $\sigma=3$ for a genuine van der Waals interaction; μ is the chemical potential and f_{HS} is the bulk free energy density of a hard-sphere system acting as a reference system; $w(\mathbf{r})$ is the fluid-fluid potential. In Eq. (9) the integrals run over the slab volume.

In order to simplify the calculations one can further simplify the continuum functional in Eq. (9) by replacing it by its lattice version. The grand potential functional for this lattice gas system then is

$$\Omega[\rho(\mathbf{r})] = k_B T \sum_{\mathbf{r} \in \Lambda} \rho(\mathbf{r}) \ln[\rho(\mathbf{r})] + [1 - \rho(\mathbf{r})] \ln[1 - \rho(\mathbf{r})] + \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}' \in \Lambda} \rho(\mathbf{r})w(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') + \sum_{\mathbf{r} \in \Lambda} [V(z) - \mu] \rho(\mathbf{r}) \quad (10)$$

where Λ is a simple cubic lattice in the region $0 \leq z \leq L$ occupied by the fluid. Here and in the following all length scales are taken in units of the lattice constant a of the order of a molecular diameter (and thus are dimensionless) so that the particle density $\rho(\mathbf{r})$ is dimensionless and varies within the range $[0,1]$. In Eq. (10) the terms in curly brackets correspond to the entropic contributions, while the other terms are directly related to the interactions present in the system. We suppose that the fluid potential $w(\mathbf{r}) = -4J(\mathbf{r})$ is given by the expression

$$J(\mathbf{r}) = J_{sr}^l \{ \delta(|\mathbf{r}|) + \delta(|\mathbf{r}|-1) \} + J^l / (1 + |\mathbf{r}|^{d+\sigma}) \theta(|\mathbf{r}|-1), \quad (11)$$

where $J(\mathbf{r})$ describes the interaction between the fluid particles, whereas the one between the fluid and the substrate particles is given by

$$J^{l,s}(\mathbf{r}) = J_{sr}^{l,s} \delta(|\mathbf{r}|-1) + J^l / |\mathbf{r}|^{d+\sigma} \theta(|\mathbf{r}|-1). \quad (12)$$

Here $\delta(x)$ is the discrete delta function and $\theta(x)$ is the Heaviside step function (with the convention $\theta(0) = 0$);

ρ_s is the number density of the substrate particles in units of a^d . Once the interaction $J^{l,s}(\mathbf{r})$ is specified, one can determine the precise form of the substrate potential

$$V(z) = -\rho_s J_{sr}^{l,s} [\delta(z) + \delta(L-z)] + v_s \left[(z+1)^{-\sigma} + (L+1-z)^{-\sigma} \right], \quad (13)$$

where

$$v_s = -4\pi^{(d-1)/2} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\sigma \Gamma\left(\frac{d+\sigma}{2}\right)} \rho_s J^{l,s}. \quad (14)$$

We consider the layers closest to the substrate to be completely occupied by the liquid phase of the fluid (which implies that we consider the strong adsorption limit), i.e., $\rho(\mathbf{r}_{\parallel}, z=0) = \rho(\mathbf{r}_{\parallel}, z=L) = 1$, which is achieved

by taking the limit $J_{sr}^{l,s} \rightarrow \infty$ and is known as (+,+) boundary conditions applied to the system under consideration.

The variation of Eq. (10) with respect to $\rho(\mathbf{r})$ leads to an equation of state for the equilibrium density $\rho^*(\mathbf{r})$. For a given geometry and surface potential $V(z)$ the solution of the equation determines the equilibrium order-parameter profile $\rho^*(\mathbf{r})$ in the system. Inserting this profile into Eq. (10) one then derives the grand canonical potential of the system and, applying the definition (1), one obtains the effective force $F_{tot}(T, \mu, L)$ between the plates of the gripper. The results are presented in the next section.

3. Evaluation of the force of attraction F_{tot} between the plates of the gripper

Evaluating the Hamaker terms A_i , A_s and $A_{i,s}$ one obtains:

$$A_l(T, \mu) = -\frac{4\pi^{(d-1)/2}}{\sigma(\sigma-1)} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\Gamma\left(\frac{d+\sigma}{2}\right)} J^l \rho_b^2(T, \mu), \quad (15)$$

$$A_s(T, \mu) = -\frac{4\pi^{(d-1)/2}}{\sigma(\sigma-1)} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\Gamma\left(\frac{d+\sigma}{2}\right)} J^s \rho_s^2(T, \mu), \quad (16)$$

$$A_{i,s}(T, \mu) = \frac{8\pi^{(d-1)/2}}{\sigma(\sigma-1)} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\Gamma\left(\frac{d+\sigma}{2}\right)} J^{l,s} \rho_b(T, \mu) \rho_s(T). \quad (17)$$

where ρ_s is the number density of the substrate, while ρ_b is the bulk limit, for given T and μ , of the number density of the fluid. Note that while $A_i > 0$, $A_s > 0$, one has $A_{i,s} < 0$, i.e. while the fluid-fluid and the direct substrate-substrate interactions contribute to a force of attraction between the plates, the fluid-substrate interaction diminishes this force acting in the opposite direction. Adding the right- and the left-hand sides of Eqs. (15), (16) and (17), one obtains the total Hamaker

term H_A , see Eq.(4), as:

$$H_A(T, \mu) = -\frac{4\pi^{(d-1)/2}}{\sigma(\sigma-1)} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\Gamma\left(\frac{d+\sigma}{2}\right)} \times [J^l \rho_b^2(T, \mu) - 2J^{l,s} \rho_b(T, \mu) \rho_s(T) + J^s \rho_s^2(T, \mu)] \quad (18)$$

$$\sim -(\alpha_s \rho_s - \alpha_l \rho_b)^2 < 0,$$

where we have taken into account that the parameters of interactions J^l , J^s , and $J^{l,s}$ can be connected [10] to the atomic polarizabilities α_l and α_s : $J^l \sim \alpha_l^2$, $J^s \sim \alpha_s^2$ and $J^{l,s} \sim \alpha_l \alpha_s$. Note that this implies

$$\frac{J^l}{\alpha_l^2} = \frac{J^s}{\alpha_s^2} = \frac{J^{l,s}}{\alpha_l \alpha_s}, \quad (19)$$

wherefrom one derives that, say, $J^s = (J^{l,s})^2 / J^l$, and thus Eq. (18) can be rewritten in the form

$$H_A(T, \mu) = -\frac{4\pi^{(d-1)/2}}{\sigma(\sigma-1)} \frac{\Gamma\left(\frac{1+\sigma}{2}\right)}{\Gamma\left(\frac{d+\sigma}{2}\right)} J^l \left[\rho_b - \frac{J^{l,s}}{J^l} \rho_s \right]^2 < 0. \quad (20)$$

The result above implies that the Hamaker term is *always negative* and, therefore, the corresponding contribution to *the force between the plates bounding the fluid will always be a force of attraction, independently on the concrete fluid and the material, from which the plates of the gripper are made, provided the both plates are from the same material.* With $d = \sigma = 3$ for real nonpolar fluids Eq. (20) further reduces to

$$H_A(T, \mu) = -\frac{\pi}{3} J^l \left[\rho_b - \frac{J^{l,s}}{J^l} \rho_s \right]^2 < 0. \quad (21)$$

If one takes, as example, such well investigated fluid as helium bounded by gold plates one can easily evaluate H_A . Using the data given in [8], one obtains that H_A is of the order of 6.8×10^{-21} J, wherefrom one concludes that, for L of the order of 10^{-8} m, the force of attraction (per unit area) between the plates of the gripper will be of the order of 1.4×10^4 N/m².

As already stated above, in addition to the attraction of the plates, which is due to the dispersion forces in the system, one also observes strong forces of attraction between them. They are due to the fluctuation of the order parameter, i.e. to the Casimir effect, see Eq. (3), and due to the competition between the liquid-like and the gas-like phases of the fluid in the capillary condensation regime, see Fig. 2 and the discussion in section III.B of Ref. [1]. An estimation of the Casimir effect can be obtained by knowing the so-called Casimir amplitude $X^{sr}(0,0) \approx -0.9$ [11], see Eq. (7), and by making use of the observation that the maximum of the Casimir force is, roughly, about 10 times deeper than this value when one scan the thermodynamic space [1]. Again, for L of the order of 10^{-8} m, as above, the numerical estimation gives that the fluctuation induced force is a force of *attraction* and is in of the order of 8×10^4 N/m² (per unit area) for a fluid with a critical temperature around the room temperatures. The dependence of this estimate on the

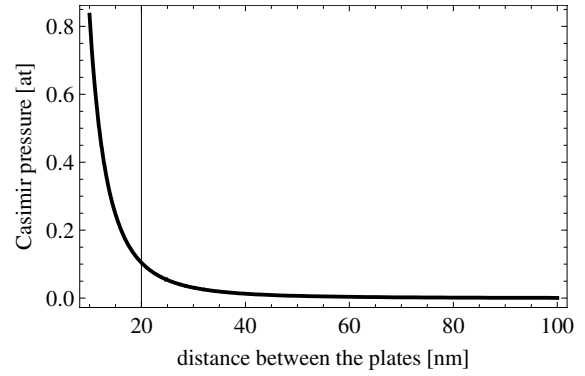


Fig. 3. The Casimir force decays with the distance as L^3 and, thus, it is easy to modify the estimations of the force given in the main text for any distance L between the plates of the gripper. The corresponding result is plotted in the current figure.

distance between the plates is illustrated in Fig. 3.

4. Discussion and concluding remarks

In the current article, we presented a model and on its basis did study the force acting between the two metallic plates of a gripper immersed in a nonpolar fluid. This force shall be taken into account when one designs the control and management of the plates of a gripper that is supposed to perform a given operation in a concrete type of fluid with a micro object. In the phase space (temperature, chemical potential), the regions where the net interaction force is the strongest have been identified. They are close to the bulk critical point of the fluid ($T = T_c, \mu = \mu_c$), and near the so-called capillary condensation regime $T < T_c$, $(L/a)(\Delta\mu/k_B T) = O(1)$, with $\Delta\mu = \mu - \mu_c < 0$. These regions shall be avoided in order to prevent sticking of the gripper plates on each other.

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